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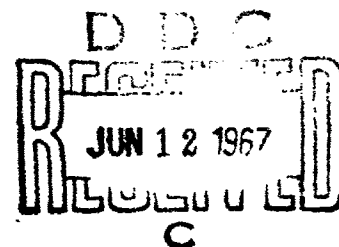
### **RANKING AND SELECTION PROCEDURES**

SHANTI S. GUPTA  
PURDUE UNIVERSITY  
DEPARTMENT OF STATISTICS  
LAFAYETTE, INDIANA

Contract No. AF 33(657)-11737

Project No. 7071

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## FOREWORD

This report was prepared by Professor Shanti S. Gupta on Contract AF 33(657)11737 for the Aerospace Research Laboratories, Office of Aerospace Research, United States Air Force. This report gives the list of publications which were partially supported under the above contract during the period July 1963-September 1966. This contract was monitored by Dr. P.R. Krishnaiah of the Applied Mathematics Research Laboratory, ARL.

# ABSTRACT

This report gives a list of publications which were partially supported under the Contract AF 33(657)11737 during the period July 1963-September 1966.

Research on ranking and selection procedures for the univariate and multivariate populations has continued apace during this period. Several associated problems of distribution theory relevant to selection and ranking problems have also been investigated. In this latter category fall the problems of distribution of the maximum of several correlated random variables and, more generally, the distribution of order statistics or a linear function of them. The application of order statistics to estimate the parameters of the logistic and the multivariate normal distribution has also been made (see nos. [7, 8, 9, 10, 24, 13, 15] in the attached list). Almost all other papers in the attached list deal with the selection and ranking problems directly. Two Ph.D. students were partially supported on this contract. Both these students finished their Ph.D. under Professor Gupta, J. Deely in 1965 and Mrs. M. Gnanadesikan in 1966.

List of Publications which were partially supported under  
the Aerospace Research Laboratories Contract AF 33(657)11737

1. Anderson, T.W. and S.M. Samuels (1967). Some inequalities among binomial and Poisson Probabilities. (to appear) Symposium on Mathematical Statistics and Probability. Univ. of California Press, Berkeley, California.
2. Barlow, R.E. and S.S. Gupta (1966). Nonparametric life test sampling plans. Technometrics, 8, 591-613.
3. Chow, Y.S., S. Moriguti, H. Robbins and S.M. Samuels (1964). Optimal selection based on relative rank (the "secretary problem"). Israel Journal of Mathematics, 2, 81-90.
4. Deely, John J. (1965). Multiple decision procedures from an empirical Bayes approach. (Ph.D. thesis). Mimeograph Series No. 45, Dept. of Statistics, Purdue University.
5. Deely, John J. and S.S. Gupta (1965). On the properties of subset selection procedures. Mimeograph Series No. 49, Dept. of Statistics, Purdue University. Submitted for publication.
6. Deely, John J. (1966). Sequential selection of the best of  $k$  populations. Mimeograph Series No. 91, Dept. of Statistics, Purdue University. Submitted for publication.
7. Gupta, S.S. (1963). Selection and ranking procedures and order statistics for the binomial distribution. Proceedings of the International Symposium on Classical and Contagious Discrete Distributions, (Ed. G.P. Patil), Pergamon Press, 1966, 219-230.
8. Gupta, S.S., K.C.S. Pillai and G.P. Steck (1964). On the distribution of linear functions and ratios of linear functions of ordered correlated normal random variables with emphasis on range. Biometrika, 51, 143-151.
9. Gupta, S.S. and B.K. Shah (1965). Exact moments and percentage points of the order statistics and the distribution of the range from the logistic distribution. Ann. Math. Statist., 36, 907-920.
10. Gupta, S.S. and K.C. Sreedharan Pillai (1965). On linear functions of ordered correlated normal random variables. Biometrika, 52, 367-379.
11. Gupta, S.S. and Mrudulla N. Wakis (1965). A system of inequalities for the incomplete gamma function and the normal integral. Ann. Math. Statist., 36, 139-149.
12. Gupta, S.S. (1965). On some multiple decision (selection and ranking) rules. Technometrics, 7, 225-245.

13. Gupta, S.S. and Mrudulla Gnanadesikan (1966). Estimation of the parameters of the logistic distribution. Biometrika, 53, 565-570.
14. Gupta, S.S. (1966). On some selection and ranking procedures for multivariate normal populations using distance functions. Multivariate Analysis, (Ed. P.R. Krishnaiah), Academic Press.
15. Gupta, S.S., A.S. Qureishi, and B.K. Shah (1967). Best linear unbiased estimators of the parameters of the logistic distribution using order statistics. (to appear) Technometrics.
16. Gupta, S.S. and W.J. Studden (1967). On some selection and ranking procedures with applications to multivariate populations. (to appear) S.N. Roy Memorial Volume, (Ed. R.C. Bose), Univ. of North Carolina Press.
17. Gupta, S.S. and Klaus Nagel (1966). On selection and ranking procedures and order statistics from the multinomial distribution. Mimeograph Series No. 77, Dept. of Statistics, Purdue University. Accepted for publication in Sankhya.
18. Gupta, S.S. and W.J. Studden (1966). Some aspects of selection and ranking procedures with applications. Mimeograph Series No. 81, Dept. of Statistics, Purdue University.
19. Holmes, Paul T. (1966). Semi-regular functions in Markov chains. Mimeograph Series No. 62, (Ph.D. thesis), Dept. of Statistics, Purdue University.
20. Nagel, Klaus (1966). On selecting a subset containing the best of several discrete distributions. Mimeograph Series No. 66, Dept. of Statistics, Purdue University. Submitted for publication.
21. Samuels, S.M. (1965). On the number of successes in independent trials. Ann. Math. Statist., 36, 1272-1278.
22. Samuels, S.M. (1966). On a Chebyshev-type inequality for sums of independent random variables. Ann. Math. Statist., 37, 248-259.
23. Samuels, S.M. (1966). Randomized rules for the two-armed bandit with finite memory. Mimeograph Series No. 71, Dept. of Statistics, Purdue University, Submitted for publication.
24. Shah, B.K. (1966). On the bivariate moments of order statistics from a logistic distribution and applications. Ann. Math. Statist., 37, 1002-1010.
25. Studden, W. J. (1966). On selecting a subset of k populations containing the best. Mimeograph Series No. 89, Dept. of Statistics, Purdue University, Submitted for publication.

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Papers and Talks Presented at the Various Professional Meetings

1. Invited paper on the Selection and ranking procedures and order statistics for the binomial distribution presented at the International Symposium on Classical Contagious and Discrete Distributions, held at McGill University, Montreal, Canada, August, 1963. (See No. 7 in the list).

2. Invited paper On the distribution of linear functions and ratios of linear functions of ordered correlated random variables (joint with K.C.S. Pillai and G.P. Steck) at the annual meetings of the American Statistical Association, Cleveland, Ohio, September, 1963. (See No. 8 in the list).

3. Contributed paper on the Estimation of the parameters of the logistic distribution (joint with Gnanadesikan) at the IMS regional meeting in Manhattan, Kansas, May 1964. (See No. 13 in the list).

4. Invited paper on Some multiple decision (selection and ranking) rules, at the joint European meetings of the IASPS, IMS and the Biometric Society, Bern, Switzerland, September 1964. (See No. 12 in the list).

5. Invited paper on Some selection and ranking procedures for multivariate normal populations using distance functions at the International Symposium on Multivariate Analysis, held at Dayton, Ohio, June, 1965. (See No. 14 in the attached list).

6. Invited talk based on the paper Best linear unbiased estimators of the parameters of the logistic distribution using order statistics, (joint with Qureishi and Shah) at the annual meetings of the ASA and IMS, Philadelphia, Sept. 1965. (See No. 15 in the list).

7. Invited talk on the joint paper with Deely On the properties of subset selection procedures at the annual meetings of the ASA and the IMS at Philadelphia, Sept. 1965. (See No. 5 in the list).



8. Invited paper On some selection and ranking problems at the Annual Army Research Conference on Design at Newark, N.J., October 1965.

9. Invited paper on Some selection and ranking procedures with application to multivariate populations (joint with W.J. Studden) at the IMS Regional meeting in Lafayette, Indiana, March 1966. (See No. 16 in the list).

10. Invited paper on Selection procedures for multivariate normal populations in terms of measures of dispersion (joint with M. Gnanadesikan) at the Annual IMS and AMS meetings at Rutgers, New Brunswick, N.J., September 1966.

1, 4, 5, 6, 8, 9 presented by Gupta

2 presented by co-author Pillai

7 presented by co-author Deely

10 presented by co-author Gnanadesikan

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